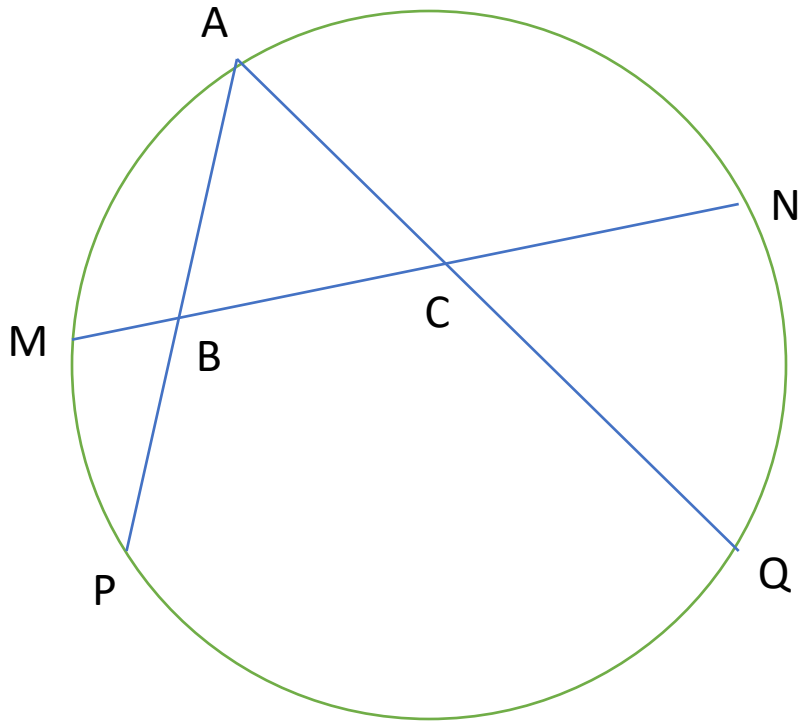
Problem Solving 8

Lecture 19 May 23 , 2021

- Q1. Suppose a_1 is a natural number. For $n > 1$ define a_{n+1} to be the largest prime number dividing a_n+1. We say a_1 is good if the sequence $a_1, a_2, a_3, \ldots$ is eventually periodic. Which of the following statements is correct:

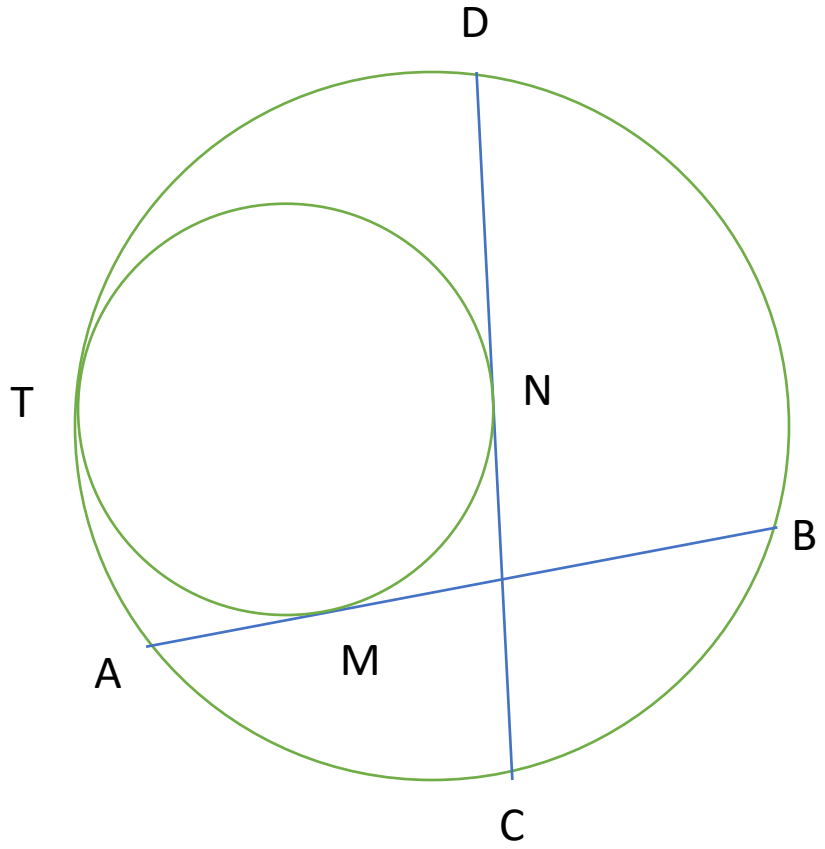
- There are only finitely many good numbers.
- There are infinitely many bad numbers
- All numbers are good
- All numbers are bad
- There are bad numbers, but only finitely many

- Q2. In the picture below, ABC is an equilateral triangle, |MB|=2, |PB|=3, and |NC|=4. Find |CQ|.



- Q3. 100 students have participated in 7 exams. In every exam, no two students have received the same score. Every student who has ranked first in one the exams, or has ranked in top 6 in 4 of the exams will be awarded a fellowship. At most how many students will be awarded a fellowship?

- Q4. In the picture below, suppose the arcs AB and BD are 60 degrees and C is in the middle of the arc AB. The smaller circle is tangent to the larger circle, AB, and CD at the points T, M, and N. Find the angle $\angle MTN$.



- Q5. Five points with integer coordinates in the plane are given. Which of the following statements about the middle points of the line connecting these points is correct?

- They all have integer coordinates
- None of them necessarily has integer coordinates
- At least one of these points has integer coordinates (but not necessarily more)
- At least two of these points has integer coordinates (but not necessarily more)
- At least three of these points has integer coordinates (but not necessarily more)

- Q6. For how many $n \leq 2021$, $n^3 + 2n^2$ is a perfect square?

- Q7. In how many ways we can color each square of a 3x3 square table with two colors (two coloring schemes are the same if they can be related by a rotation).